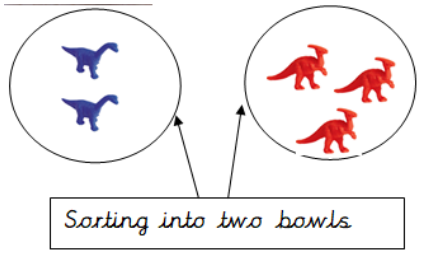
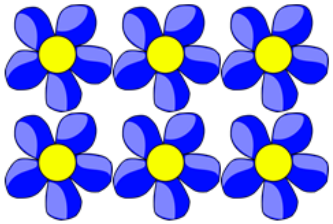
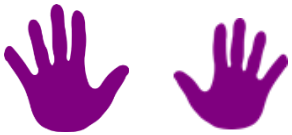
Addition

Foundation Stage 1 Objectives:

Birth –to 11 months – notice changes in number of objects/images, sounds in groups of and up to 3
8 – 20 months - has some understanding that things exist even when out of sight
16-26 months – Begins to organise and categorise objects -sorting
22 – 36 months – knows that a group of things changes in quantity when something is added
30 – 50 months – separates a group of 3 or 4 objects in different ways, beginning to recognise that the total is still the same
In practical activities and discussions begins to use the vocabulary involved in addition

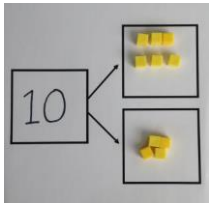
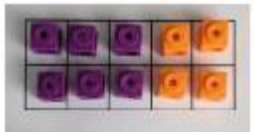
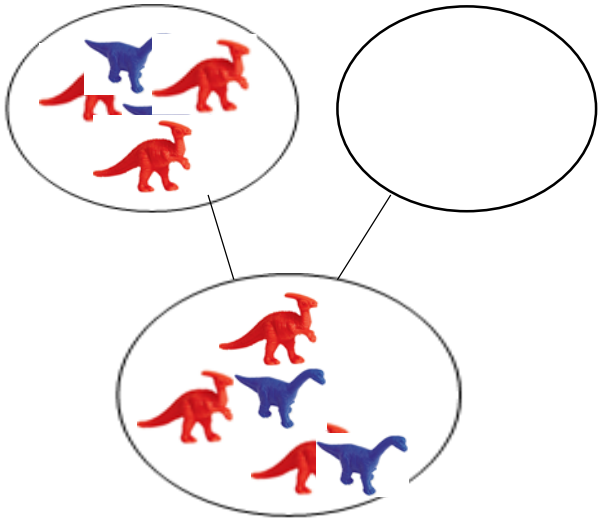
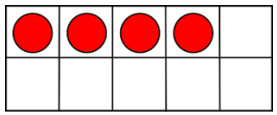
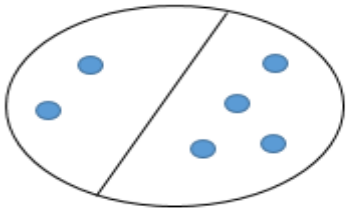
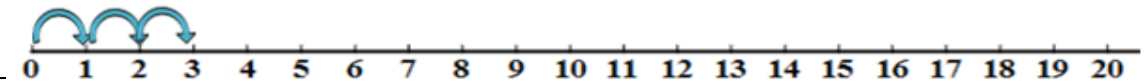
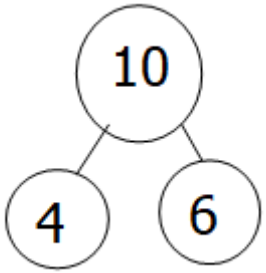
Concrete	Pictorial	Abstract
Nursery rhymes and number stories. Using numbers and objects in the environment   Counting using hands and through movement. Counting using concrete objects for 1 to 1 correspondence as well as for grouping and partitioning   	I can count …  	Writing the digit to represent the quantity 6

Foundation Stage 2 Objectives:

40-60 months – finds the total number of items in two groups by counting all of them

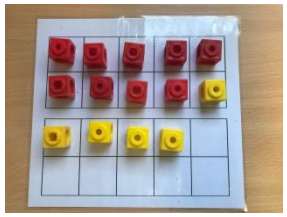
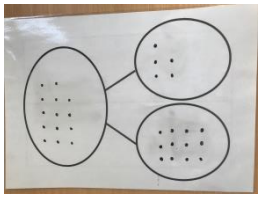
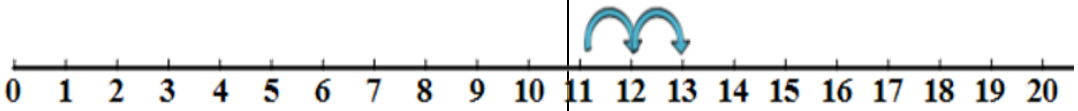
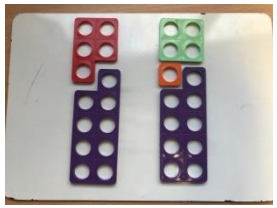
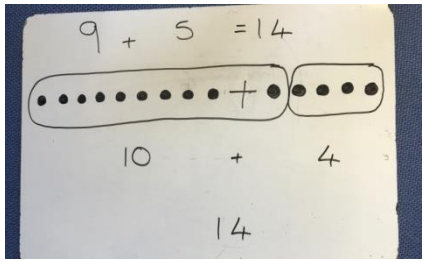
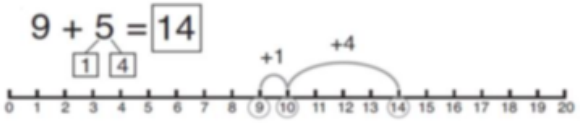
Early Learning Goal - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Count on from first group to add two groups of objects

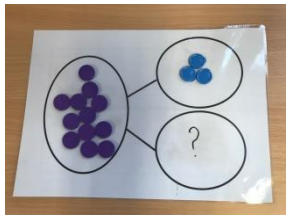
Concrete	Pictorial	Abstract
   Use objects and maths resources to add two numbers together as a group.   +   	Use pictures to add 2 numbers together    4 + ☐ = 6 2 + 4 = 6 	Children will annotate their pictures with number sentences.   4 + 2 = 6

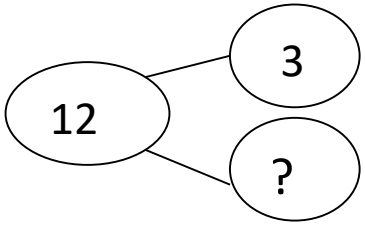
Year 1 Objectives:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add one-digit and two-digit numbers to 20, including
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as
- 17 = □ – 9

Concrete	Pictorial	Abstract
 Using resources to investigate the creation of numbers up to 20. First steps to bridging. Using place value – counting on in ones, using a number line, bead string and 100 square etc.	Drawing images to reflect concrete representations  Start with the larger number and count on. 	Writing the number sentences to support the pictorial 9 + 5 = 14
Progressing onto using knowledge of numbers bonds within 10 when crossing the tens boundary e.g. 9 + 5 = Start with the 9, then add 1 to make 10, then add the remaining 4. 	Use pictures or a number line. Regroup or partition the smaller number to make 10.  	Place the larger number in your head and count on the smaller number to find the answer. **5 + 9 = 14** **9 + 1 = 10** **10 + 4 = 14** If I have nine, how many more do I need to make 10? How many more do I add on now? 15 +1 = 16 16 = 15 + 1 (commutative law)

Finding missing numbers.





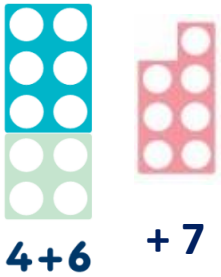
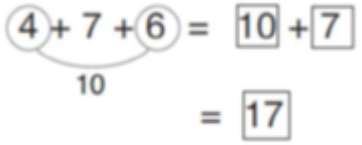
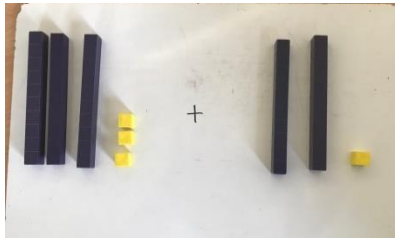
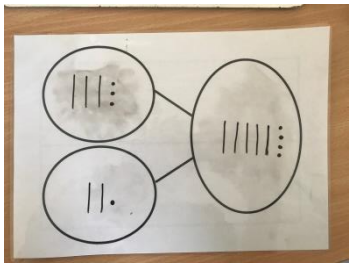
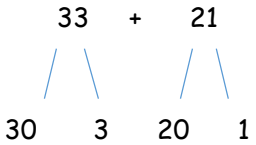
3 + ☐ = 12

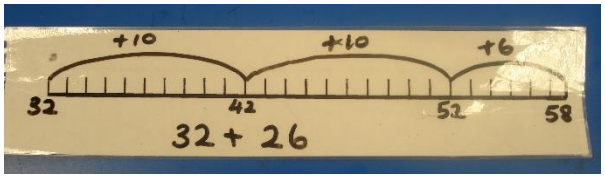
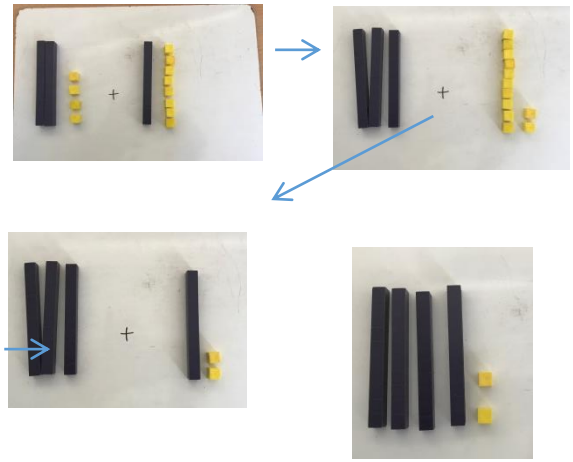
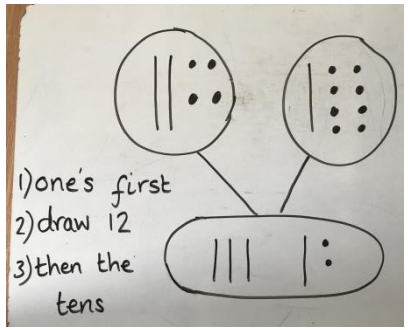
☐ + 6 = 15

Year 2 Objectives:

- solve problems with addition:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- add numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and 1s
  - a two-digit number and 10s
  - 2 two-digit numbers
  - adding 3 one-digit numbers
- show that addition of 2 numbers can be done in any order (commutative)

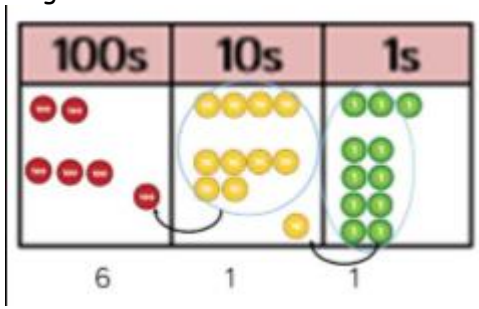
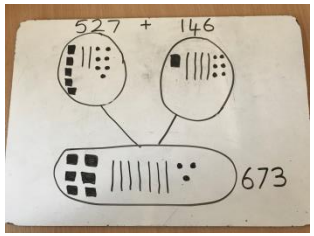
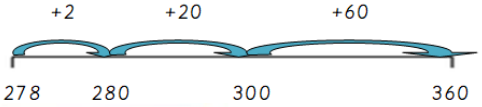
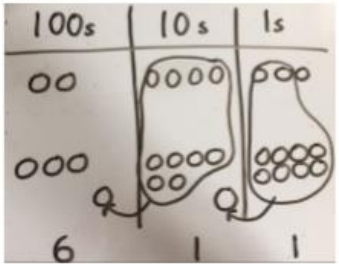
recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

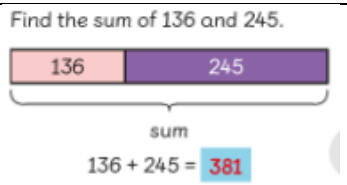
Concrete	Pictorial	Abstract
4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Partitioning both numbers into tens and ones 33 + 21 = 54 OR 21 + 33 = 54 	Start with the two parts and combine to create the whole (Representing the concrete). 	33 + 21 = 30 + 20 = 50 3 + 1 = 4 50 + 4 = 54 

Adding the second number to the first by partitioning the tens and ones, using a variety of resources		32 + 26 = 58 32 + 20 = 52 52 + 6 = 58 32 + 26 32 20 6
Use manipulatives to secure understanding of crossing 10's boundaries. 24 + 18 = 42 Add together the ones first then add the tens. Develop to include regrouping. 		Progress onto the expanded written column method, with column headings. Up to 2 by 2 digits without crossing the 10s boundary initially but achieving by the end of the year. T O 2 4 + 1 8 1 2 3 0 4 2

Year 3 Objectives:

- add numbers mentally, including:
  - a three-digit number and 1s
  - a three-digit number and 10s
  - a three-digit number and 100s
- add numbers with up to 3 digits, using formal written methods of column addition
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition

Concrete	Pictorial	Abstract
Use e.g. base ten, place value counters. Begin in the ones column. For every 10 created exchange for a 10 counter.  As children move on to decimals and money, decimal place value counters can be used to support learning.	Draw images to represent concrete resources:  These informal representations may be used to clarify understanding and can be used alongside number lines. It will also aid fluency in mental calculations.   The bar model can reinforce the concept of part part whole.	Expanded formal written method with labelled columns and starting with the ones column, progressing from: 1. No crossing of boundaries 2. Crossing the tens or hundreds boundary in 3 digit number 3. A combination of the above. 475 + 267 = 742 H T O 400 70 5 + 200 60 7 100 10 700 40 2 =742 H T O 4 7 5 + 2 6 7 1 1 **7 4 2** Progressing on to compact column method.



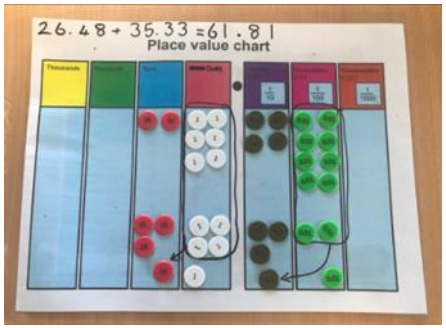
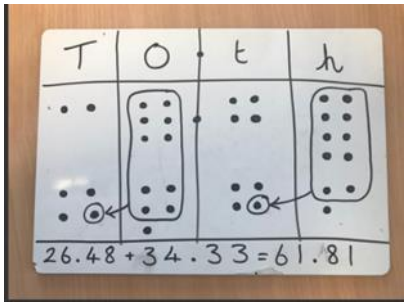
Year 4 Objectives:

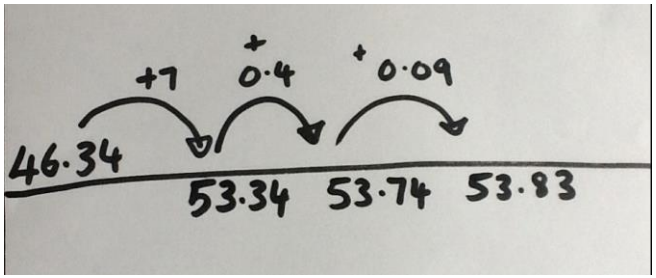
- add numbers with up to 4 digits using the formal written methods of column addition where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why

Concrete	Pictorial	Abstract
See above.	See above.	

A line to be left to record numbers crossing boundaries either above or below the answer. To be consistent at your school (numbers written smaller).

	Th	H	T	O
	4	3	6	5
+	2	5	8	6
	1	3	7	4

		Formal written method with columns labelled Progressing from adding 2 four digit numbers to adding 3 four digit numbers, not exceeding the thousands column. Build knowledge by crossing one boundary at a time, beginning with the tens. Develop understanding by moving onto crossing multiple boundaries, not following a set pattern. Variation – missing numbers. 1 2 1 **8 3 2 5** Th H T O 4 3 6 5 + 2 5 7 6 1 1 **6 9 4 1**
	 There were 6000 books for sale at a book fair. 3419 books were sold on the first day of the fair and 2268 books were sold on the second day. How many books were left at the end of the second day? 6000 3419 2268 ?	When secure, progress to adding money with two decimal places. Place value headings to be labelled. Th H T O 2 9 . 4 5 + 2 5 . 2 9 1 . 1 £ **5 4 . 7 4**



To promote fluency number lines can be used for addition of decimals

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{rcrl} 20 & + & 5 & \\ \underline{40} & + & \underline{8} & \\ 60 & + & 13 & = 73 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 536 \\ +\,85 \\ \hline 621 \\ \scriptstyle 1\,1 \end{array}$$

$$\begin{array}{r} 72.8 \\ +\,54.6 \\ \hline 127.4 \\ \scriptstyle 1\,1 \end{array}$$

$$\begin{array}{rr} & £23.59 \\ + & £7.55 \\ \hline & £31.14 \\ & \scriptstyle 1\,1\qquad 1 \end{array}$$

$$\begin{array}{rr} & 23.361 \\ & 9.080 \\ & 59.770 \\ + & 1.300 \\ \hline & 93.511 \\ & \scriptstyle 2\,1\qquad 2 \end{array}$$

Year 5 Objectives:

- add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Concrete	Pictorial	Abstract
See above	Where necessary do not be afraid to use the expanded method for initial explanation. Number lines promote fluency and are a clear assessment tool for teachers. Bar models to be used to support their understanding of problems – help them identify what they need to do.	Addition of 4 and 5 digit numbers to one million. TTh Th H T O 2 9 6 1 5 + 2 5 4 3 9 4 3 5 2 3 1 1 1 **9 8 5 7 7** Children provided with numbers where they have to decide whether to 'carry' or not – this will identify if they have a secure understanding of place value. Pupils to record numbers using commas e.g. 98,577 Addition of numbers with 2 decimal places in context e.g. money and measurement. H T O . t h 2 3 8 . 8 4 + 4 2 6 . 5 2 1 1 . **£ 6 6 5 . 3 6** Pupils to use estimation before completing calculations. Provide examples in context to support understanding of the importance of estimating. Use rounding to estimate. E.g.

		423 + 158 + 296 = Estimate: 420 + 160 + 300 =

Year 6 Objectives:

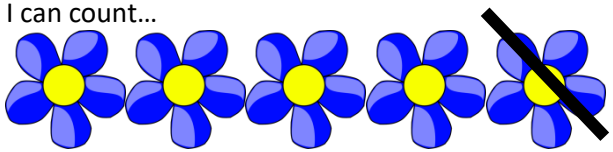
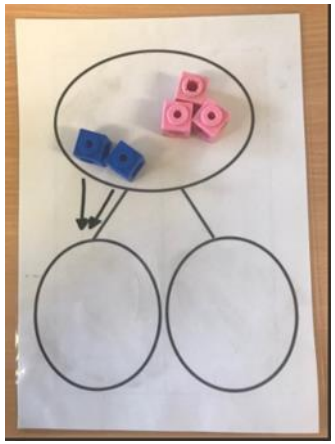
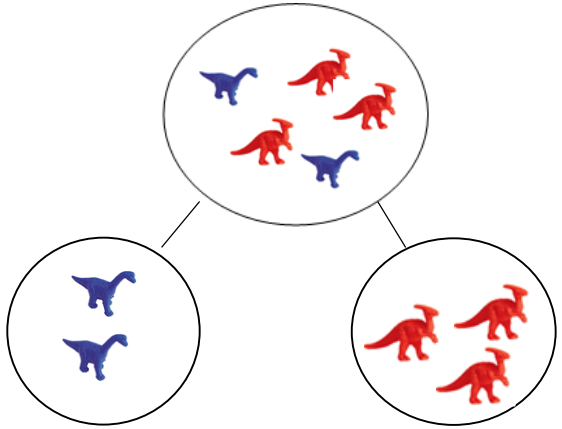
- perform mental calculations, including with mixed operations and large numbers
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Concrete	Pictorial	Abstract
See above for concrete examples to use when beginning addition work to emphasise the need to exchange when you make ten and place value understanding.	**See above for pictorial images to use when beginning addition work.**	Pupils to record numbers using commas e.g. 2,598,577 Addition of numbers, not exceeding 10 million. 1 2 6 3 4 3 + 2 8 7 3 5 2 3 2 2 1 5 4 1 1 1 **7 3 5 8 4 9** Addition of numbers with up to 3 decimal places, using 0 as a place holder. 1 . 8 2 1 + 0 . 4 3 3 2 . 0 2 6 1 . 1 **4 . 2 8 0**

Subtraction

Foundation Stage 1 Objectives:

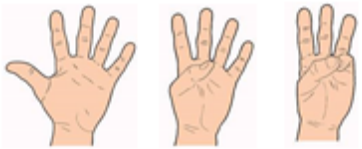
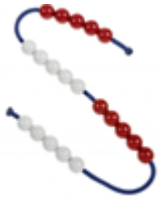
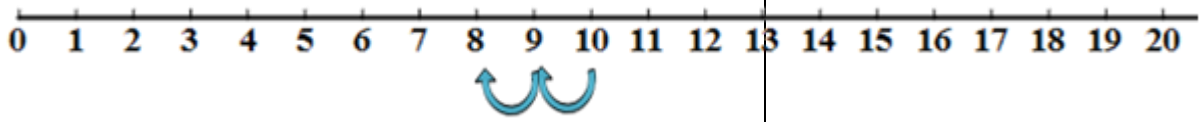
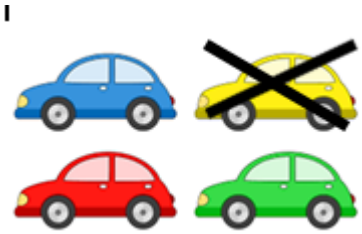
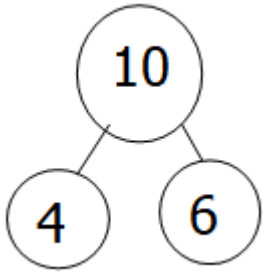
- Birth –to 11 months – notice changes in number of objects / images, sounds in groups of and up to 3
- 8 – 20 months - has some understanding that things exist even when out of sight
- 16-26 months – Begins to organise and categorise objects -sorting
- 22 – 36 months – knows that a group of things changes in quantity when something is added or taken away
- 30 – 50 months – separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same

Concrete	Pictorial	Abstract
Use a variety of contexts, such as nursery rhymes to give purpose to the resources you use. Use of objects in the environment – remove one to show how to 'take away'. 	I can count... 	The use of nursery rhymes to count backwards in steps of one. Counting back verbally – 5, 4, 3, 2, 1… in the context of stories.
Being able to separate objects and know the total is still the same. 		5 apples take away two apples leaves 3 apples. Starting to look at the abstract. 5 – 2 = 3

Foundation Stage 2 Objectives:

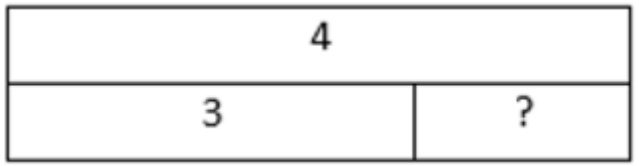
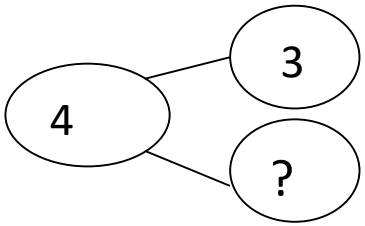
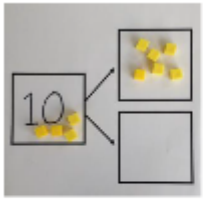
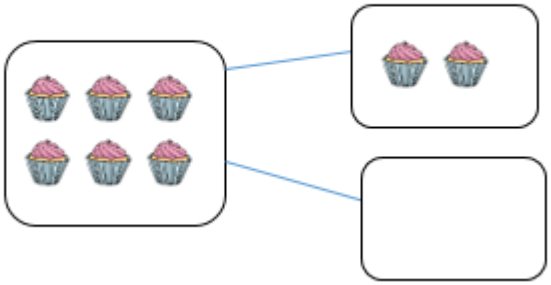
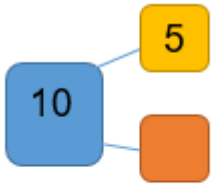
- 40-60 months – Understands subtraction as taking away objects from a group and counting on how many are left.

- In practical activities and discussions begin to use the vocabulary involved in addition and subtraction.
- Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer

Concrete	Pictorial	Abstract
Subtraction using concrete objects. Hide or take away with the focus being 1 less before counting back.   Count back using out hands   (6 - 2 = 4) 	Number line, with steps recorded below  Pictorial representation with crossing out to show 1 less. E.g. 1 car left the car park… 	Recording number sentences after practical activities and discussions 10 - 4 =6 

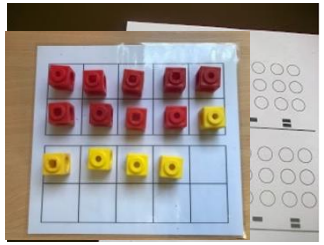
Year 1 Objectives:

- read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9

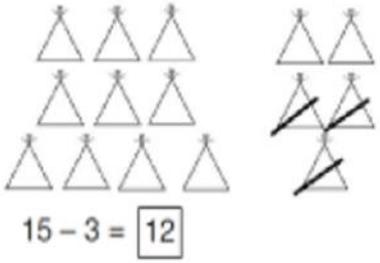
Concrete	Pictorial	Abstract
Early in Year 1, use resources such as numicon to show the whole and part. 	**Include place value headings in line with your school.** 	4 – 3 = 1 
 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

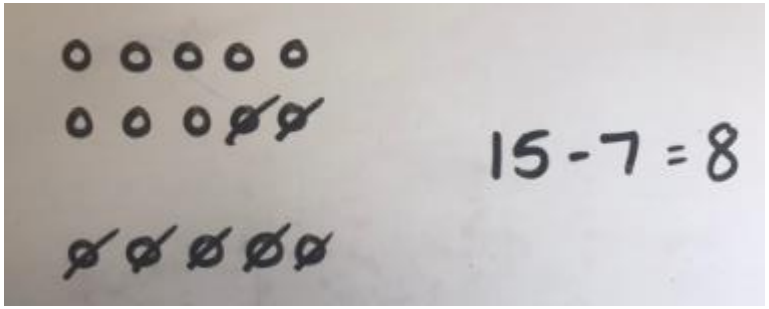
Begin with subtraction of numbers, initially with no exchange.
Make the larger number with beads, then move beads along your string as you count back.
13 – 4 =

Use resources such as tens frame and number beads to model elements of subtraction e.g. 'crossing the tens' boundary, counting back in ones.



Cross out drawn objects to show what has been taken away.

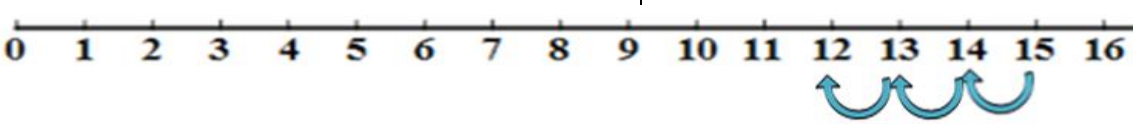




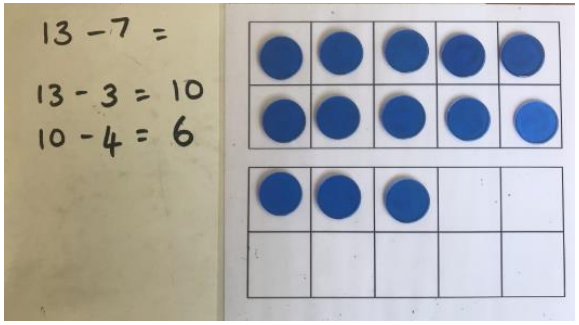
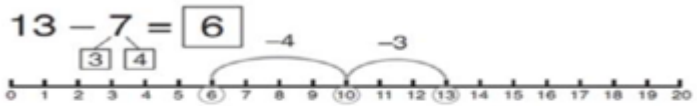
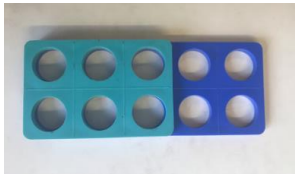
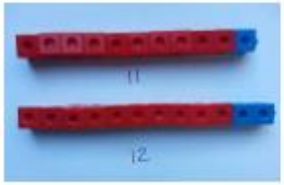
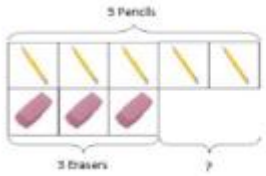
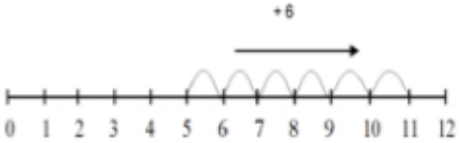
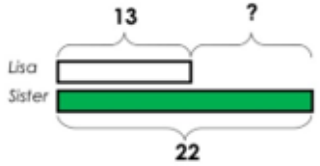
Introduce children to problem solving using missing number problems:

15 - 3 = ☐
15 - ☐ = 12
☐ - 12 = 3
☐ - ☐ = 12

Put 15 in your head, count back 3. What number are you at? Use your fingers to help.

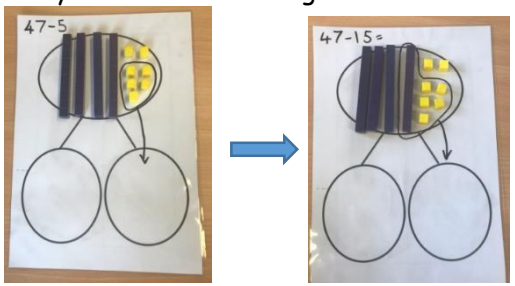
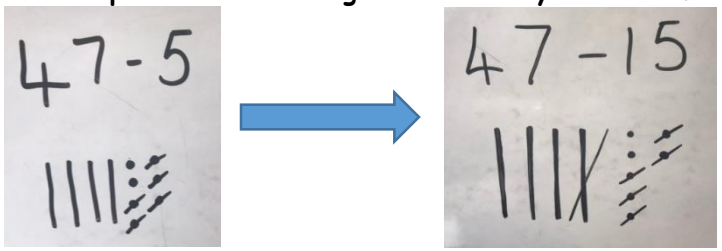
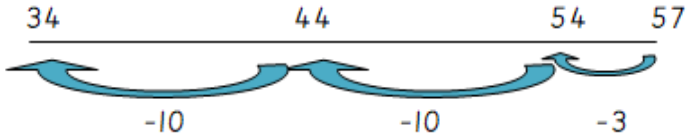


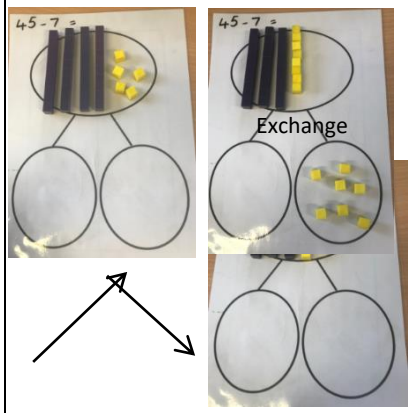
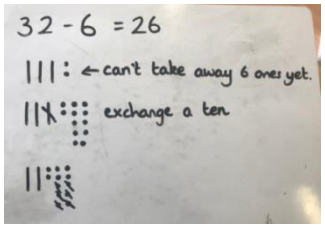
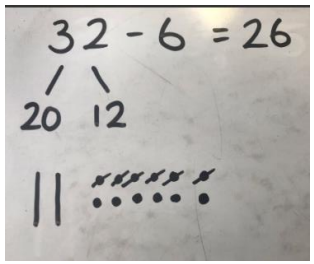
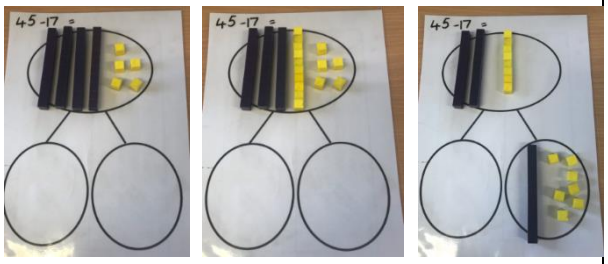
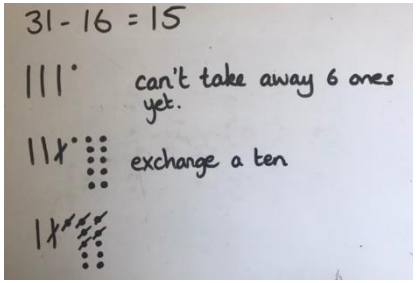
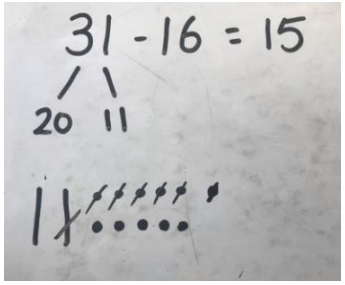
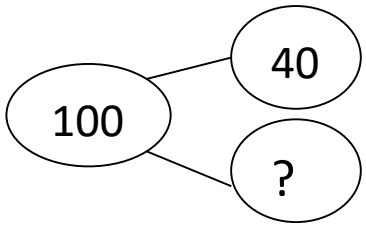
15-7 = 8

Children practise partitioning the number they are subtracting into parts which help bridge the 10. 	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	13 – 7 = How many do we take off to reach 10? 13 – 3 = 10 How many do we have left to take off? 10 – 4 = 6
Use numicon to find the difference between numbers. e.g. The difference between 10 and 6.  Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 22 shells; Helen has 13 shells. Find the difference between the numbers of shells. 22 – 13 = 9

Year 2 Objectives:

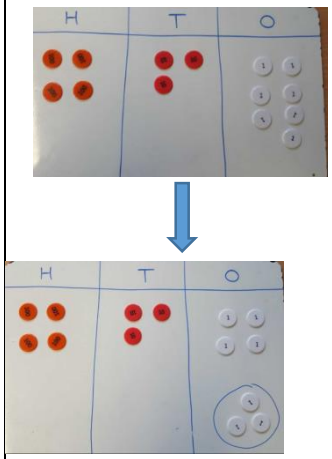
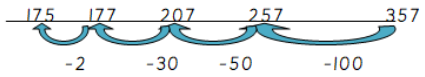
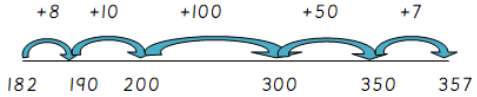
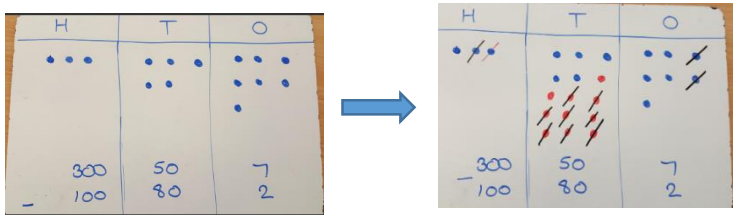
- solve problems with subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

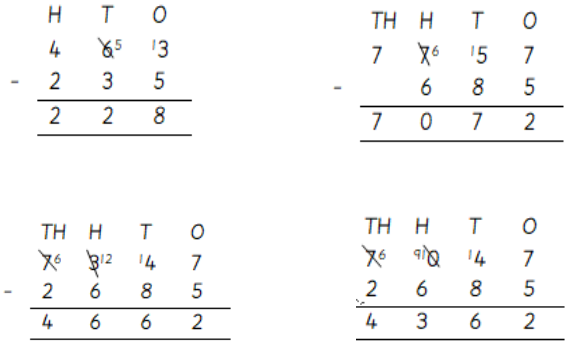
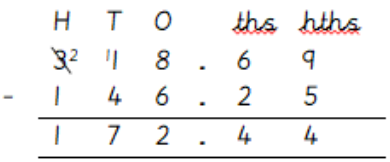
Concrete	Pictorial	Abstract
Subtract a single digit from a two digit, initially without an exchange. 	**Include place value headings in line with your school.**  Progress to subtraction of two digits, without exchange. Progress on to counting back/subtraction using an unmarked number line, when place value is secure : E.g. 57 – 23 = 34 	47 – 5 = 42 47 – 15 = 32

Progressing to an exchange. Create your number Carry out the subtraction 	We can either parition the number we are subtracting or the nuimber we are subtracting from. e.g. partition the 32 into 20 and 12 or 22 and 10  Or  Use part, part, whole to support the concept of regrouping. Number lines can be used once the place value understanding is secure.	45 - 7 = 38
Two digit subtract two digit, with an enxchange.  Create your number Exchange Carry out the subtraction	  Use part, part, whole to support the concept of regrouping. Number lines can be used once the place value understanding is secure.	31 – 16 = 15
Use part, part, whole frames to illustrate that addition and subtraction are inverse calculations – used for missing number problems.	Use part, part, whole and bar models to illustrate and secure the structures of the mathematics.  100 60 	40 + 60 = ☐ 100 - ☐ = ☐ 60 = 100 - ☐

Year 3 Objectives:

- subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

Concrete	Pictorial	Abstract
Building on Year 2, using concrete and pictorial resources before progressing to formal columnar methods, initially using the expanded form to secure place value. 	Using number lines to subtract and count on using 3 digit numbers 357 – 182 = 175  Encourage children to use their knowledge of number to partition in order to subtract larger numbers. Using counting on: 357 – 182 = 175  	Extended written method: 324 – 161 200 ~~300~~ 120 4 100 60 1 100 60 3 = 163 Leading to: T O 40 7 - 20 3 20 4 Repeat for HTO – TO, leading onto compact method, **ONLY** if pupils are very secure Only extending to compact if very secure. H T O → H T O 100 70 6 → 1 7 6

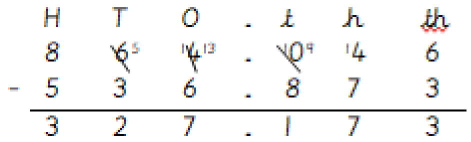
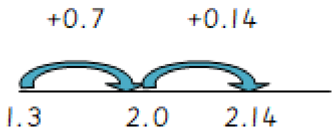
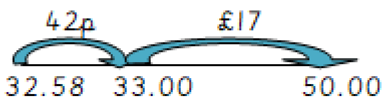
		- 60 4 = 100 10 2 - 64 = 112
Year 4 Objectives: • Subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.		
Concrete	Pictorial	Abstract
Build on previous year group models and images.	Continue to explore formal columnar written method and how to exchange in order to calculate. Begin with 3 digit subtract 3 digit; moving to 4 digit subtract 3 digit and then 4 digit subtract 4 digit. At each stage, only make one exchange initially.	Continue to explore formal columnar written method and how to exchange in order to calculate. Begin with 3 digit subtract 3 digit; moving to 4 digit subtract 3 digit and then 4 digit subtract 4 digit. At each stage, only make one exchange initially. Begin to include 0 as a place holder: model how to exchange. 
Progress to subtraction of numbers with 2 decimal places in context £318.69 - £146.25 = £172.44 Estimate answers before calculation e.g. 318.69 – 146.25 =	Use pictorial representations as shown above where appropriate.	

320 – 150 = 170		

Year 5 Objectives:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

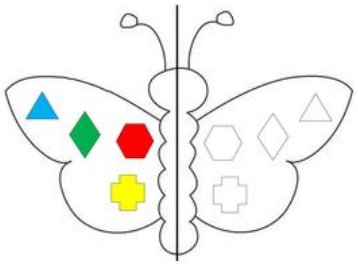
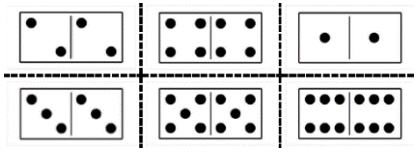
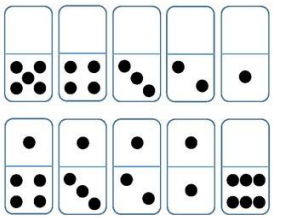
Concrete	Pictorial	Abstract
Continue to build on Year 4 before subtracting with more than 4 digits, including numbers with differing decimal places e.g 134.25 – 23.4 =		Subtracting 5 digit numbers, moving towards 6 digit numbers and using 0 as a place holder. Discrete teaching of the requirement to make more than one exchanges must be taught, when dealing with 0. TTH TH H T O 4 6 7̶⁶ ¹0̶⁹ ¹4 - 2 3 4 5 8 2 3 2 4 6 Model how to use 0 as a place holder when calculating with numbers with different decimal places. H T O . t h ³4̶ ¹6 7 . ³4̶ ¹0 - 2 8 4 . 2 5 1 8 3 . 1 5

Year 6 Objectives: • solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why		
Concrete	Pictorial	Abstract
Apply knowledge and understanding to the solving of different problems involving subtraction dealing with digits to 1,000,000. Subtract numbers with up to 3 decimal places, in context such as measure.		
	Use counting on to subtract smaller numbers with decimals. 2.14 – 1.3 = 0.84  Use counting on to subtract money from multiples of 10 e.g. £50. £50 – 32.58 = £17.42 	Solve problems in real contexts e.g. A car company needed to sell 345,234 cars in 3 months. In the first month they sold 122,408 and in the second month they sold 159,386 cars. How many did they need to sell in the third month? 345,234 – (122,408 + 159,386) =63,440

Multiplication

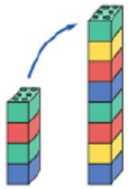
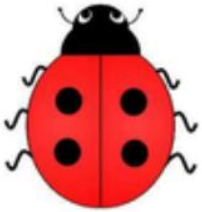
Foundation Stage 2 Objectives:

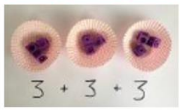
- 40 – 60 months - Finds the total number of items in two groups by counting all of them.
- In practical activities and discussions begins to use the vocabulary involved in multiplication
- Early Learning Goal - They solve problems, including doubling, halving and sharing.

Concrete	Pictorial	Abstract
Looking at reflections in the mirror Make prints by folding paper in half 		
Doubling on hands and finding doubles on dominoes etc. 		 Match the dots/colour them in…

Year 1 Objectives:

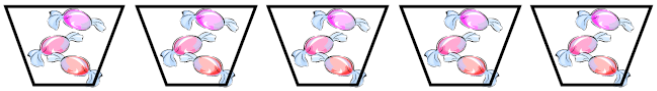
- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Non-statutory guidance: Through grouping small quantities, pupils begin to understand: multiplication and doubling numbers and quantities.
- They make connections between arrays, number patterns, and counting in 2s, 5s and 10s.

Concrete	Pictorial	+
 Start with doubling using concrete resources	 Use diagrams to show doubling.	2+2=4
  Count in 2s, 5s and 10s using resources to support	 Count in 2s, 5s and 10s on your hands and recognise the patterns on number lines. 	2,4, 6 etc.

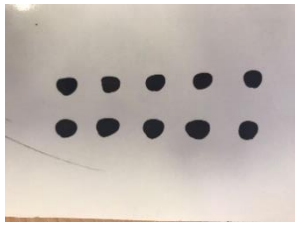


Introduce repeated addition for multiplication. Use resources to show the amount in each group. Progress on to representing this as an array. Use contextual links to problem solve.

Solve multiplication problems through repeated addition, using pictures, diagrams and own drawings to support understanding.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Starting to use arrays and looking for patterns when counting in multiples.

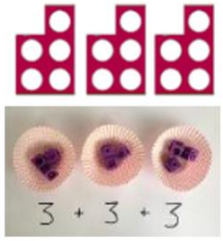
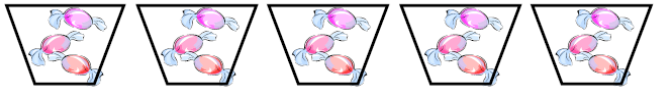
5+5+5 = 15

Year 2 Objectives:

Pupils should be taught to:

- Count in steps of 2, 3, 5 and 10.
- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs

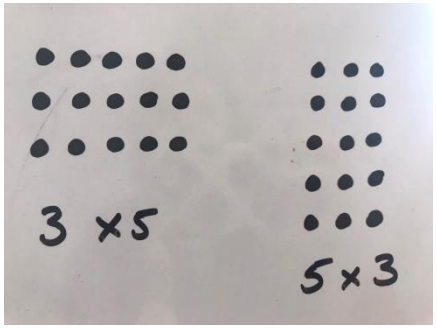
- show that multiplication of 2 numbers can be done in any order (commutative)
- solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

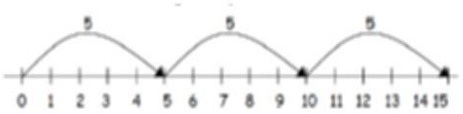
Concrete	Pictorial	Abstract
  Count in 2s, 5s and 10s using resources to support	 Count in 2s, 5s and 10s on your hands and recognise the patterns on number lines. 	2,4, 6 etc.
 		**5+5+5 = 15** **3 x 5 = 15** **5 x 3 = 15** **(commutativity)** **Relate to division facts (once division has been taught):** **15÷3= 5** **15÷5= 3** **<u>Variation Ideas:</u>** 2 x 3 2 x 30 2 x 300 20 x 3 200 x 3

Introduce repeated addition for multiplication. Use resources to show the amount in each group. Progress on to representing this as an array. Use contextual links to problem solve.

Solve multiplication problems through repeated addition, using pictures, diagrams and own drawings to support understanding when **grouping.**

Show repeated addition as jumps on a number line.





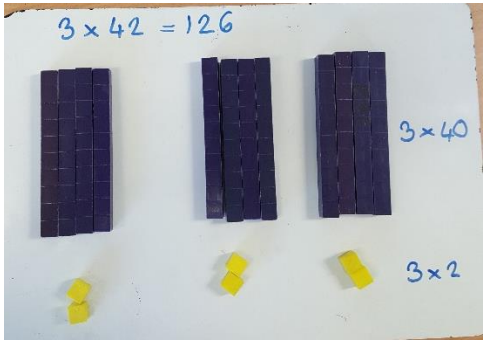
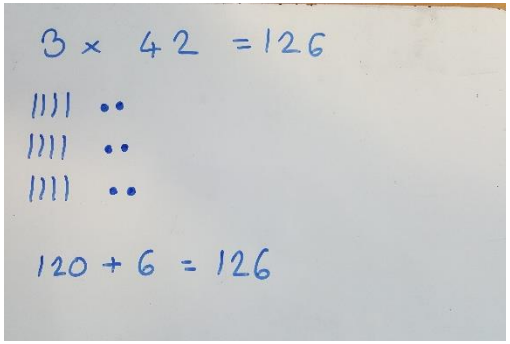
5 + 5 + 5 = 15

5	5	5

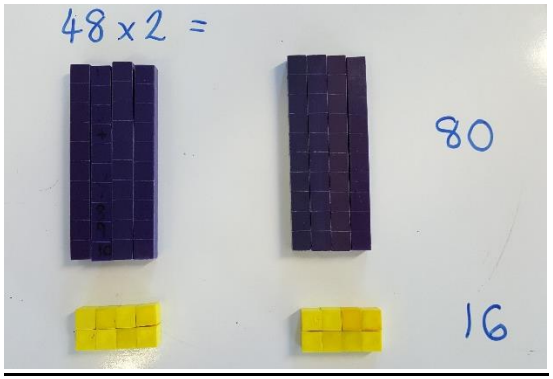
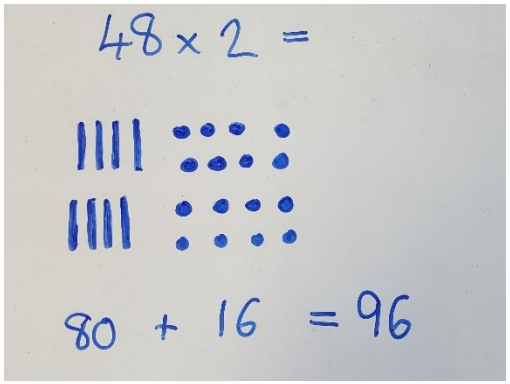
Year 3 Objectives:

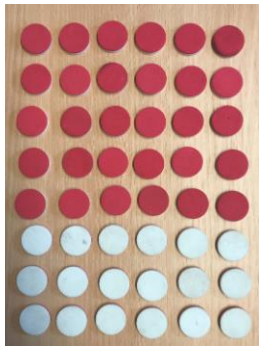
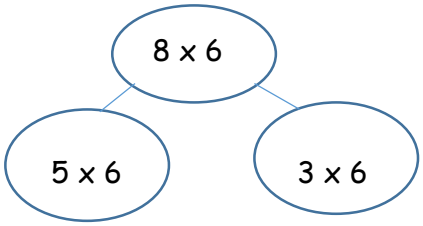
Pupils should be taught to:

- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

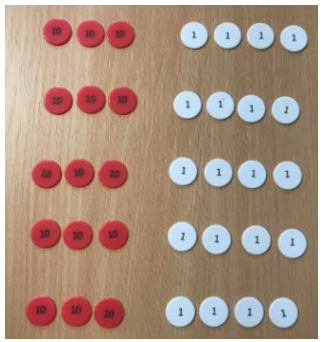
Concrete	Pictorial	Abstract
 Build on use on arrays to show the commutative law.	5 x 3 = 15 3 x 5 = 15  ***'The product of ___ and ___ is equal to the product of ___ and ___.'*** This can then be simplified to: ***'___ times ___ is equal to ___ times ___.'***	5x3=15 3x5=15 **Relate to division facts:** **15÷3= 5** **15÷5= 3**
		3 x 42 = 126 3 x 40 = 120 3 x 2 = 6 120 + 6 = 126

x	**40**	**2**	=
3	120	6	126

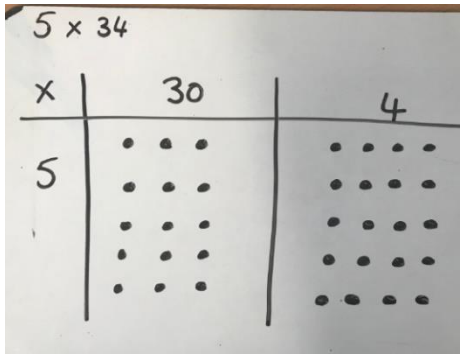
Doubling  	Doubling  	Doubling 48 x 2 = 96 48 (40 x 2) + (8 x 2) 80 + 16 = 96

Year 4 Objectives: Pupils should be taught to: • recall multiplication facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together 3 numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects		
Concrete	Pictorial	Abstract
See above for arrays to demonstrate commutativity.	***'The product of ___ and ___ is equal to the product of ___ and ___.'*** This can then be simplified to: ***'___ times ___ is equal to ___ times ___.'***	
Using partitioning of a factor to support mental approaches with multiplication 	 Discussion point: Which other ways could you partition the factors? e.g. 4 x 6 and 4 x 6 8 x 3 and 8 x 3 8 x 5 and 8 x 1 Could also been shown with a numberline	8 x 6 = 5 x 6 = 30 3 x 6 = 18 30 + 18 = 48

5 x 34 =



34	34	34	34	34



5 x 34 =
5 x 30 = 150 because 5 x 3 = 15
5 x 4 = 20
150 + 20 = 170

x	30	4	=
5	150	20	170

150 + 20 = 170

34 x 5 =
5 x 30 = 150
5 x 4 = 20
150 + 20 = 170

	3	4
x		5
	2	0
1	5	0
1	7	0

This may lead to a compact method.

	3	4
x		5
	2	
1	7	0

Progress onto 3 digit multiplied by a 1 digit number using the same strategies as above.

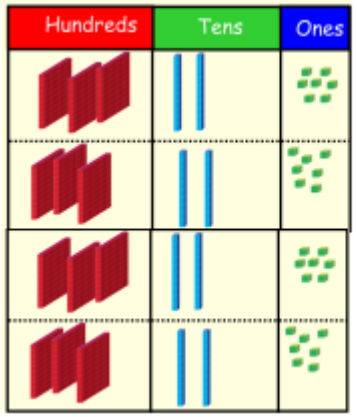
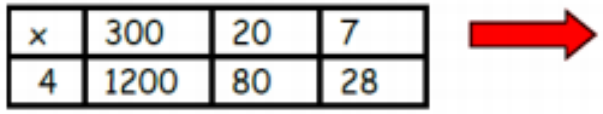
Demonstrate 3 x 1 digit before using compact method.

274 x 8 =
8 x 200 = 1600
8 x 70 = 560
8 x 4 = 32
1600 + 560 + 32 = 2192

Year 5 Objectives:

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply numbers mentally, drawing upon known facts
- multiply whole numbers and those involving decimals by 10, 100 and 1,000

Concrete	Pictorial	Abstract
Children can continue to be supported by place value counters at this stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first.	 	Secure compact multiplication with 3 x 1 digit and 4 x 1 digit. 3 2 7 x 4 2 8 8 0 1 2 0 0 1 3 0 8 Leading to a compact method: 3 2 7 x 4 1 2 1 3 0 8

Multiplying 2 x 2 digit using the expanded method.

Demonstrate using the grid method 2 x 2 digit before moving to a more formal method to secure understanding.

x	30	6	
20	600	120	= 720
4	120	24	= 144

720 + 144 = 864

	3	6	
x	2	4	
	2	4	(4 x 6)
1	2	0	(4 x 30)
1	2	0	(20 x 6)
6	0	0	(20 x30)
8	6	4	

Extending onto compact multiplication before moving onto 3 and 4 digit numbers x 2 digit.

Progress onto calculations with missing numbers.

Leading to:

	3	6
x	2	4
	2	
1	4	4
1		
7	2	0
8	6	4

Year 6 Objectives:

Pupils should be taught to:

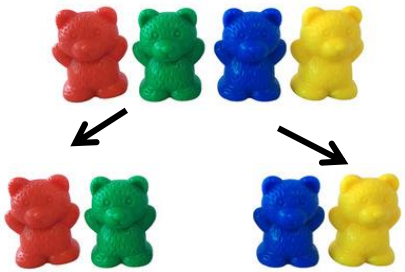
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers

Concrete	Pictorial	Abstract
As year 5 but progressing onto using decimals TO.t x O as an expanded calculation. (tens, ones and tenths x ones) If pupils are secure, they may prgress onto the compact method.		Expanded T O . t 2 3 . 3 x 7 2 . 1 2 1 . 0 1 4 0 . 0 1 6 3 . 1 Compact T O . t 2 3 . 3 x 7 2 2 1 6 3 . 1

Division

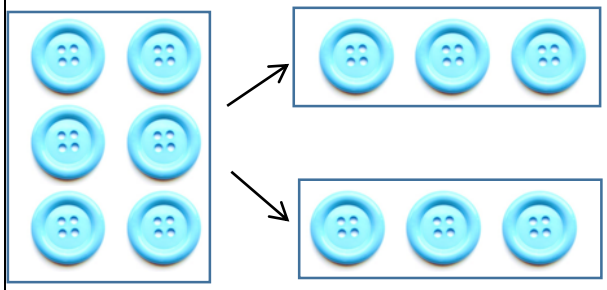
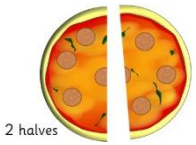
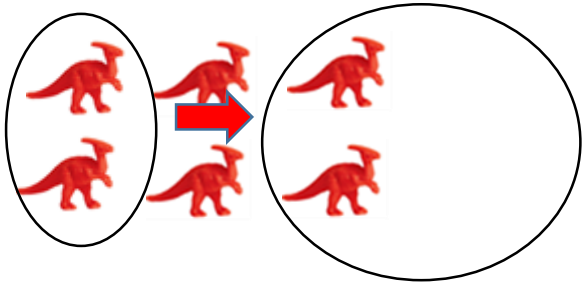
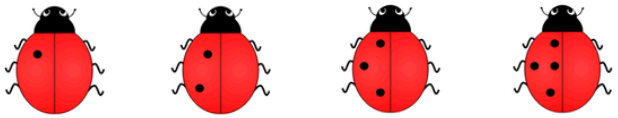
Foundation Stage 1:

- 30 – 50 months - Separate a group of three or four objects in different ways, beginning to recognise the total is still the same.

Concrete	Pictorial	Abstract
Separate groups of objects in different ways – begin to introduce half/double if pupils are ready. 		

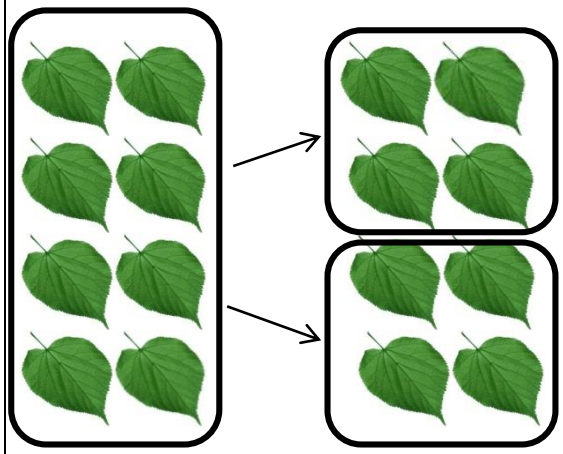
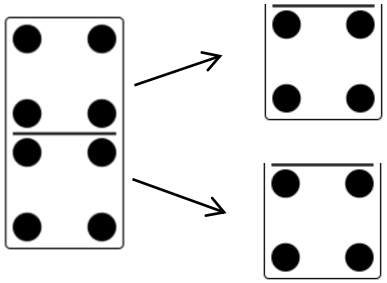
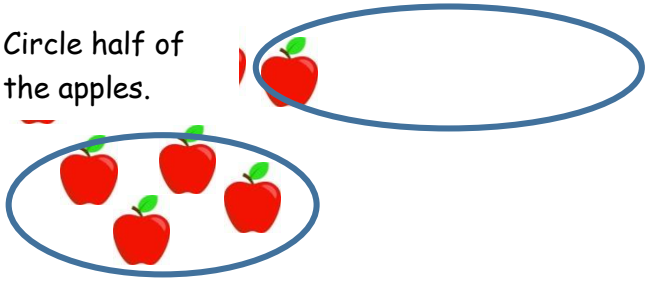
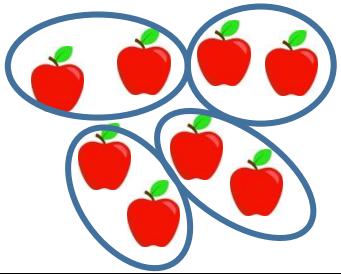
Foundation Stage 2 Objectives:

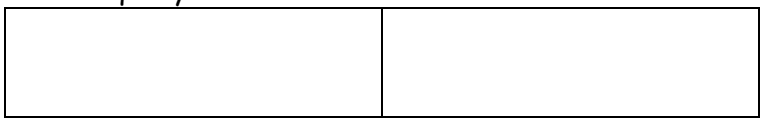
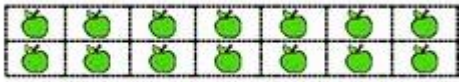
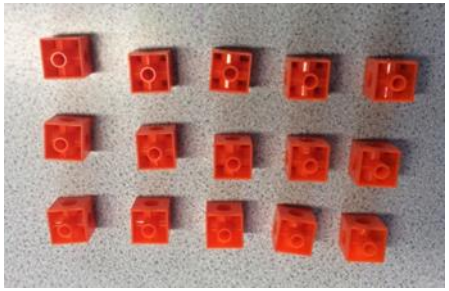
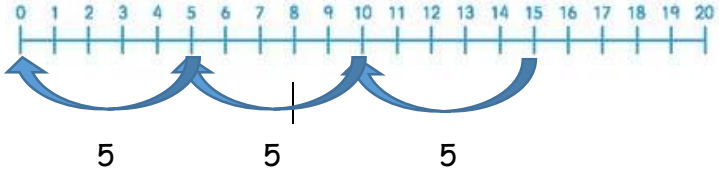
- 40 – 60 months - They solve problems, including doubling, halving and sharing.

Concrete	Pictorial	Abstract
Practically halving everyday objects – the halves being the same size. Begin with halving play dough and other items that could be cut, then use hoops /halving mats etc. to separate items.  	Halving images  Finding the other half of everyday shapes to match them e.g. cups, beans	Half of … is … (adult written)
Doubling everyday items e.g. compare bears, counters etc. 	Doubling e.g. the spots on the ladybird. 	Double 1 is 2…(adult written) 1 + 1 = 2 2 + 2 = 4

Year 1 Objectives:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

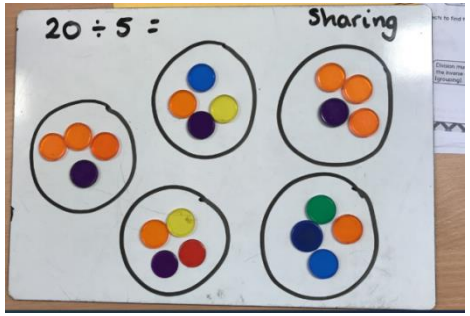
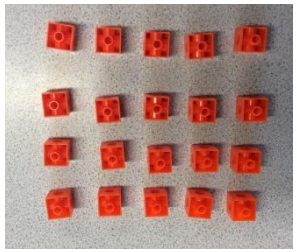
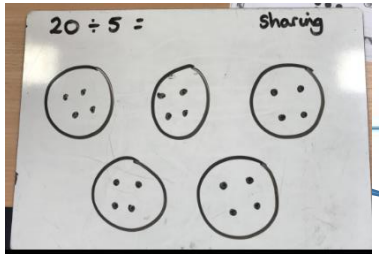
Concrete	Pictorial	Abstract
Find half of even numbers up to 12, using fingers and objects. 		Half of 8 is 4 8 ÷ 2 = 4
Develop finding half of numnbers before moving onto finding quarters whole  1/2s  1/4s 	Circle half of the apples.  Circle a quarter of the apples. 	Half of 8 is 4. 8 ÷ 2 = 4 One quarter of 8 is 2 8÷ 4 = 2

Begin to find half of a quantity using sharing e.g. half of 14 cubes by sharing one at a tie into two sorting dishes. 	Share equally between 2.  	Half of 14 is 7 14 shared between 2 is 7.
Grouping: Use concrete and visual arrays/sets of objects to find answers to e.g. 15 girls play a game in teams of 5. How many groups are there? 	 5  5  5  5 5 5	Total number of objects ÷ number in each group = number of groups. There are 3 groups of 5 in 15, so 15 ÷ 5 = 3

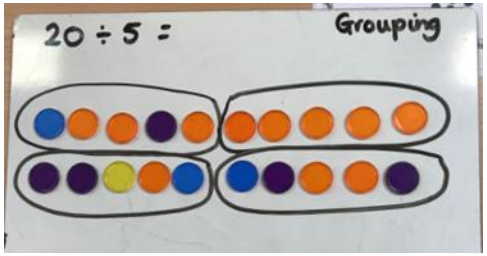
Year 2 Objectives:

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs
- show that multiplication is commutative but division is not
- solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

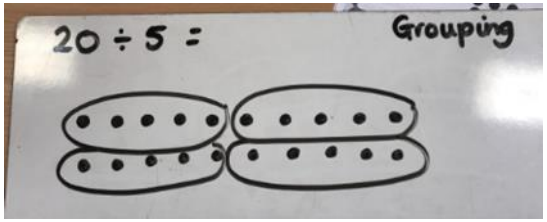
Concrete	Pictorial	Abstract
Continue to explore division as sharing  20 shared between 5 groups gives us 4 in each group. 	Show sharing in 'chunks'   5 5 5 5 20 divided by 5 equals 4 rows. 	20 ÷ 5 = 4 

Grouping

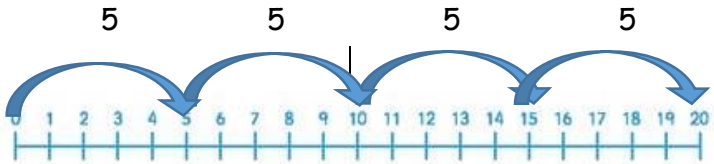


How many groups of 5 make 20?

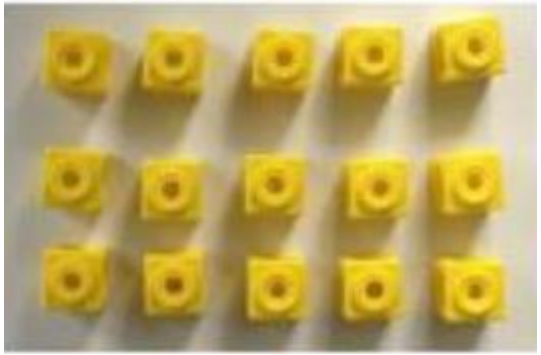
20 has been divided into 4 equal groups of 5.



20 ÷ 5 = 4



Link division to multiplication by creating an array and finding 4 realted number sentences.



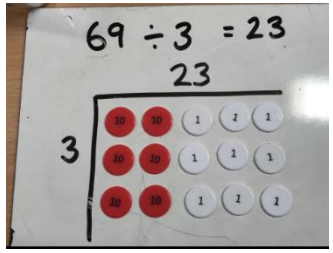
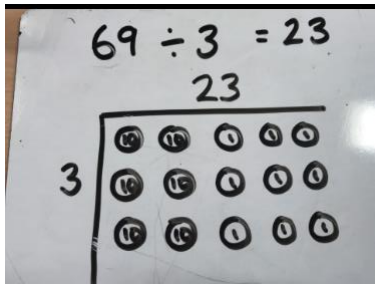
15 ÷ 3 =

15 ÷ 3 = 5
15 ÷ 5 = 3
3 x 5 = 15
5 x 3 = 15

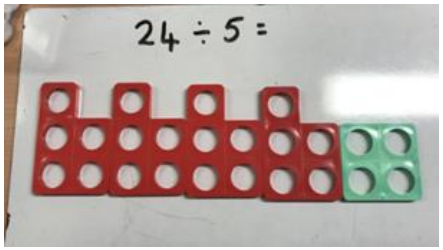
Year 3 Objectives:

Pupils should be taught to:

- recall and use division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

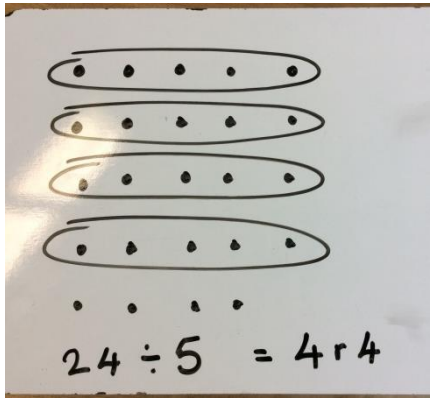
Concrete	Pictorial	Abstract
Pupils to understand that division is not commutative. Use the relationship of multiplication facts to calculate.	See above for examples of grouping and sharing using concrete and pictorial resources, and exploring the relatrionship between multiplication and division.	
Pupils begin to explore formal written method, at first with no remainders. 		69 ÷ 3 = 23 2 3 3 | 6 9

Progress onto division with remainders, within the ones column so there is no need to exchange when subtracting using a more formal method.



or

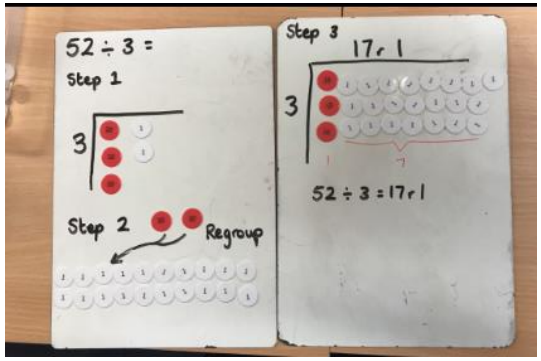
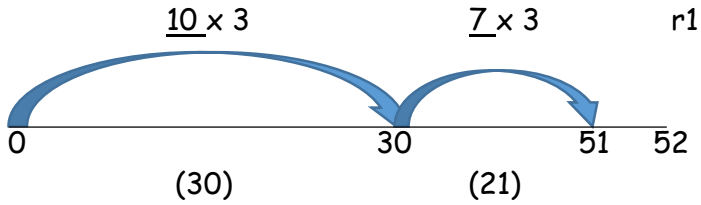
50 ÷ 3 =



24 ÷ 5 = 4 r4

	1	6 r2	
3	5	0	
-	3	0	(10x)
	2	0	
-	1	8	(6x)
		2	

(No exchange required for the subtraction)

Year 4 Objectives: Pupils should be taught to: • recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to divide mentally, including dividing by 1 • solve problems involving dividing a two digit, then three-digit number by one-digit number using a formal layout		
Concrete	Pictorial	Abstract
As above and developing written method with the need to exchange for 2 digit numbers divided by 1 digit. 	52 ÷ 3 = 17 r1 	1 7 r1 3 | 5 2 - 3 0 (10x) 2 2 - 2 1 (7x) 1
Progress onto division of 3 digit by 1 digit		1 3 1 r3 4 | 5 2 7 - 4 0 0 (100x) 1 2 7 - 1 2 0 (30x) 7 - 4 (1x) 3

Year 5 Objectives: Pupils should be taught to: • identify multiples and factors, including finding all factor pairs of a number, common factors of two numbers, know and use the vocabulary of prime numbers and establish whether a number up to 100 is prime • divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • divide whole numbers and those involving decimals by 10, 100 and 1000		
Concrete	Pictorial	Abstract
Use concrete and pictorial strategies as shown above if pupils require continued support with their understanding.		
Divide 4 digit numbers by 1 digit using a short division and where appropriate, begin to interpret remainers as fractions. Pupils begin to look at and discuss decimals in relation to money.		Pupils supported with multiplication where appropriate by writing the times table at the side of their work. $\begin{array}{r} 1\ 3\ 1\ 5\ \text{r}3 \\ 4\overline{)\,5\ {}^{1}2\ 6\ {}^{2}3} \end{array}$ Working towards $\begin{array}{r} 1\ 3\ 1\ 5\ \frac{3}{4} \\ 4\overline{)\,5\ {}^{1}2\ 6\ {}^{2}3} \end{array}$ Pupils encouraged to simplify the remaining fraction where possible. $\begin{array}{r} 1\ 3\ 1\ 5.\ 7\ 5 \\ 4\overline{)\,5\ {}^{1}2\ 6\ {}^{2}3.\ {}^{3}0\ {}^{2}0} \end{array}$

Year 6 Objectives: Pupils should be taught to: • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context		
Concrete	**Pictorial**	**Abstract**
Pupils use long division to calculate 3 or 4 digit numbers by 2 digit numbers. Progress to interpreting the remainder as a decimal, where appropriate within the context of the problem.		The multiplication table to be recorded next to the question. 14 28 42 56 70 84 98 112 27 7/14 = 27 ½ = 27.5 15 30 45 60 75 90 105 120